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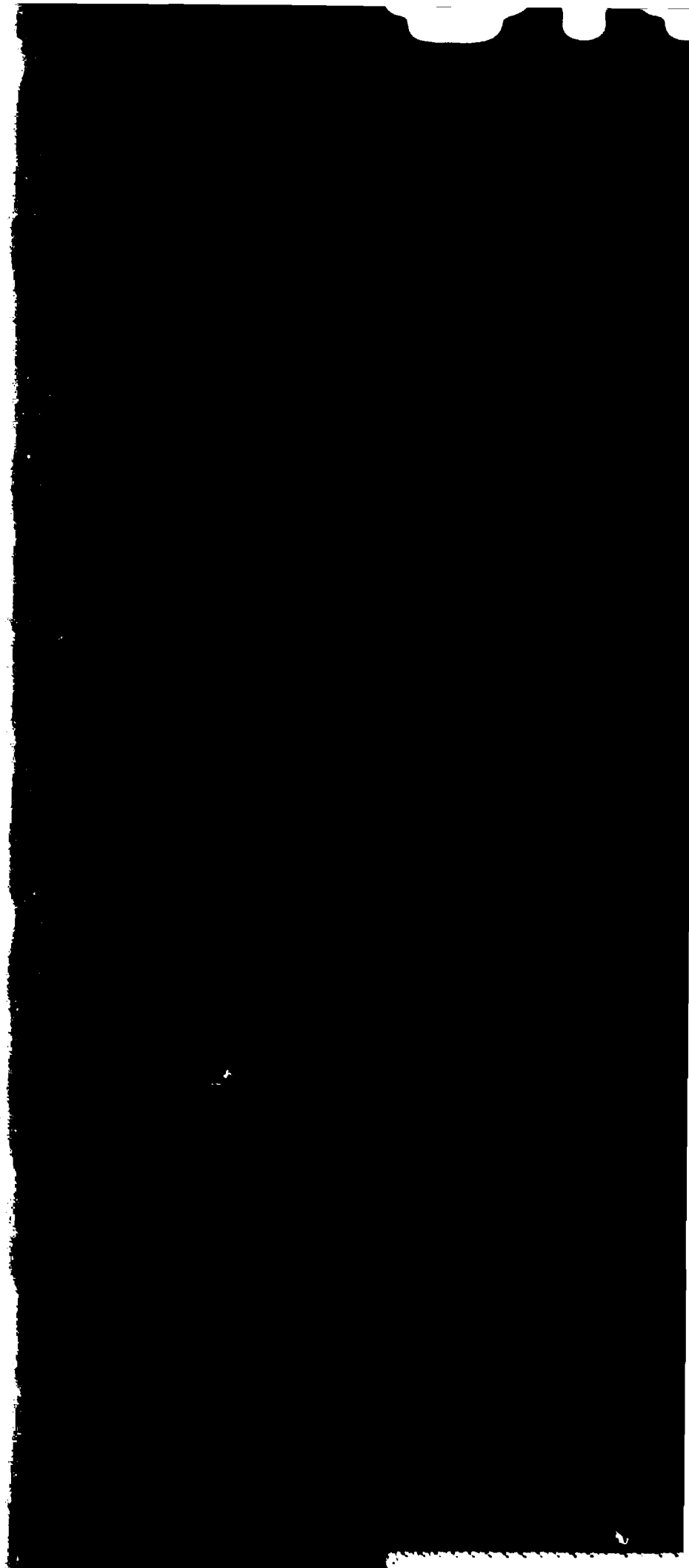
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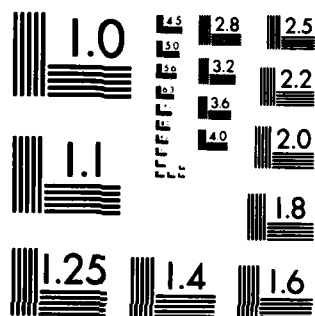
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AXIOMATIC CHARACTERIZATIONS OF CONTINUUM STRUCTURE FUNCTIONS*

AD-A149 817

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ABSTRACT

A continuum structure function is a nondecreasing mapping from the unit hypercube to the unit interval. Axiomatic characterizations of the continuum structure functions based on the Barlow-Wu and Natvig multistate structure functions are derived.

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1. INTRODUCTION

Let $C = \{1, 2, \dots, n\}$ denote a set of components and let $\Delta = [0, 1]^n$. A nondecreasing mapping $\gamma: \Delta \rightarrow [0, 1]$ with $\gamma(\underline{0}) = 0$ and $\gamma(\underline{1}) = 1$ is said to be a continuum structure function (CSF). If $\sup_{\tilde{X} \in \Delta} [\gamma(1_i, \tilde{X}) - \gamma(0_i, \tilde{X})] > 0$ for each $i \in C$, where (δ_i, \tilde{X}) denotes $(X_1, \dots, X_{i-1}, \delta, X_{i+1}, \dots, X_n)$, γ is said to be weakly coherent.

Definition

Let P_1, \dots, P_r denote the r minimal path sets of a binary coherent structure function. If

$$\gamma(\tilde{X}) = \max_{1 \leq j \leq r} \min_{i \in P_j} X_i \quad (\tilde{X} \in \Delta),$$

γ is said to be a Barlow-Wu CSF [2].

Definition

Let $\{\phi_\alpha, 0 < \alpha \leq 1\}$ be a class of binary coherent structure functions such that $\phi_\alpha(Y_\alpha)$ is a left-continuous and non-increasing function of α for fixed \tilde{X} where $Y_{\alpha i}$ is the indicator of $\{X_i \geq \alpha\}$, $i=1, 2, \dots, n$. If

$$\gamma(\tilde{X}) \geq \alpha \text{ iff } \phi_\alpha(Y_\alpha) = 1 \quad (\tilde{X} \in \Delta, 0 < \alpha \leq 1),$$

γ is said to be a Natvig CSF [3].

In this paper, we present axiomatic characterizations of the Barlow-Wu and Natvig CSFs. In particular, we show that γ is a Barlow-Wu CSF if and only if it satisfies the following conditions:

- C1 γ is continuous
- C2 $P_\alpha \neq \emptyset$ and $P_\alpha \subset \{0, \alpha\}^n$, $0 < \alpha \leq 1$
- C3 There is no nonempty open set $A \subset \Delta$ such that γ is constant on A
- C4 γ is weakly coherent

where $P_\alpha = \{\tilde{X} \in \Delta \mid \gamma(\tilde{X}) \geq \alpha \text{ whereas } \gamma(\tilde{Y}) < \alpha \text{ for all } \tilde{Y} < \tilde{X}\}$ and where $\tilde{Y} < \tilde{X}$ means that $\tilde{Y} \leq \tilde{X}$ but that $\tilde{Y} \neq \tilde{X}$.

Some consequences of these axioms are deduced in Section 2, and in Section 3 we present our main results: an axiomatic characterization of the Barlow-Wu CSF and an analogous characterization of the Natvig CSF. Our approach was suggested by the Borges-Rodrigues characterizations of the Barlow-Wu and Natvig multistate structure functions [5] though, as we show in Section 4, their characterizations are incorrect.

2. SOME DEDUCTIONS FROM THE AXIOMS

Let $U_\alpha = \{\tilde{X} \in \Delta \mid \gamma(\tilde{X}) \geq \alpha\}$ and $L_\alpha = \{\tilde{X} \in \Delta \mid \gamma(\tilde{X}) \leq \alpha\}$, $0 \leq \alpha \leq 1$. Further, define $K_\alpha = \{\tilde{X} \in \Delta \mid \gamma(\tilde{X}) \leq \alpha \text{ whereas } \gamma(\tilde{Y}) > \alpha \text{ for all } \tilde{Y} > \tilde{X}\}$, $0 \leq \alpha < 1$.

Proposition 2.1

Let γ be a CSF.

- (i) γ is right (left)-continuous if and only if each $U_\alpha(L_\alpha)$ is closed.
- (ii) If γ is right (left)-continuous, then each $P_\alpha(K_\alpha)$ is nonempty and $\tilde{X} \in U_\alpha(L_\alpha)$ if and only if $\tilde{X} \geq (<) \tilde{Y} \in P_\alpha(K_\alpha)$.
- (iii) If γ is continuous, then $\gamma(P_\alpha) = \{\alpha\}$, $0 < \alpha \leq 1$, and $\gamma(K_\alpha) = \{\alpha\}$, $0 \leq \alpha < 1$.

Proof: The proofs of (i) and (iii) are straightforward; see [4] for the proof of (ii).

Proposition 2.2

If γ is a continuous CSF, conditions C2 and

$$C2' \quad K_\alpha \neq \emptyset \text{ and } K_\alpha \subset \{\alpha, 1\}^n, \quad 0 \leq \alpha < 1$$

are equivalent.

Proof: Since γ is continuous, each K_α is nonempty. We show that, if C2 holds, then $K_\alpha \subset \{\alpha, 1\}^n$ for all $\alpha \in [0, 1)$.

Suppose, conversely, that for some $\alpha \in [0, 1)$ there exists a vector $\underline{y} \in K_\alpha$ such that $\underline{y} \notin \{\alpha, 1\}^n$. Then there exists at least one component, k say, such that $y_k \notin \{\alpha, 1\}$. Either $0 \leq y_k < \alpha < 1$ or $0 \leq \alpha < y_k < 1$; we consider these two cases separately.

Suppose, firstly, that $0 \leq y_k < \alpha < 1$. By Proposition 2.1, $\gamma(\underline{y}) = \alpha$ and $\gamma(\delta_k, \underline{y}) > \alpha$ if $y_k < \delta < \alpha$. Let $\gamma(\delta_k, \underline{y}) = \xi$; then $(\delta_k, \underline{y}) \in U_\xi$. Since U_ξ is closed there exists, by Proposition 2.1, an $\underline{x} \leq (\delta_k, \underline{y})$ such that $\underline{x} \in P_\xi$. Now $\underline{y} \notin U_\xi$ and so $y_k < x_k \leq \delta$. Thus $0 \leq y_k < x_k \leq \delta < \alpha < \xi$ and so $\underline{x} \notin \{0, \xi\}^n$, in contradiction to C2.

Suppose, now, that $0 \leq \alpha < y_k < 1$. Again $\gamma(\underline{y}) = \alpha$. Let $\gamma(1_k, \underline{y}) = \delta > \alpha$. Since $\gamma(x_k, \underline{y})$ is a continuous, nondecreasing function of x for fixed (\cdot_k, \underline{y}) , it follows from the intermediate value theorem that, for given ξ with $\alpha < \xi < y_k \wedge \delta$, there exists a $w \in (y_k, 1)$ such that $\gamma(w_k, \underline{y}) = \xi$. Thus $(w_k, \underline{y}) \in U_\xi$ and hence there exists an $\underline{x} \leq (w_k, \underline{y})$ such that $\underline{x} \in P_\xi$. Now $\underline{y} \notin U_\xi$ and so $y_k < x_k \leq w$. It follows that $0 \leq \alpha < \xi < y_k < x_k \leq w$ and hence $\underline{x} \notin \{0, \xi\}^n$, in contradiction to C2.

Thus, a continuous CSF satisfying C2 also satisfies C2'. A similar argument verifies the converse. \square

Proposition 2.3

If γ is a CSF which satisfies C1, C2 and C3, then $\gamma(\{0, \alpha\}^n) = \{0, \alpha\}$ for all $\alpha \in [0, 1]$.

Proof: If $\alpha = 0$ there is nothing to prove, so suppose that, for some $\alpha \in (0, 1]$, there exists a vector $\underline{x} \in \{0, \alpha\}^n$ such that $\beta = \gamma(\underline{x}) \notin \{0, \alpha\}$. It is easily seen that $0 < \beta < \alpha$ and that $\underline{x} \neq \underline{0}$ or $\underline{\alpha}$, and hence we can write

$$x_{i_j} = \begin{cases} 0 & \text{for } j=1, 2, \dots, k \\ \alpha & \text{for } j=k+1, \dots, n \end{cases}$$

for some k with $1 \leq k \leq n-1$.

Since $\underline{x} \in U_\beta \cap L_\beta$, and both are closed, it follows from Proposition 2.1 that there exist a $\underline{z} \in P_\beta$ and a $\underline{w} \in K_\beta$ such that $\underline{z} \leq \underline{x} \leq \underline{w}$. This ordering will only hold if $\underline{z} \in \{0, \beta\}^n - \{0\}$ satisfies $z_{i_j} = 0$ for $j=1, 2, \dots, k$ and if $\underline{w} \in \{\beta, 1\}^n - \{1\}$ satisfies $w_{i_j} = 1$ for $j=k+1, \dots, n$ and so $A = (z_1, w_1) \times \dots \times (z_n, w_n) \subset \Delta$ is open. Further, since $\underline{z} \in P_\beta$ and $\underline{w} \in K_\beta$, it follows that $\gamma(\underline{x}) = \beta$ for all $\underline{x} \in A$, in contradiction to C3. Thus $\gamma(\underline{x}) \in \{0, \alpha\}$ as claimed. \square

Proposition 2.4

If γ is a CSF which satisfies C1, C2 and C3, then $P_\alpha = \alpha P_1$ for all $\alpha \in (0, 1]$.

Proof: Suppose that $\alpha < 1$, otherwise there is nothing to prove, and let $\underline{X} \in P_\alpha$ so that $\gamma(\underline{X}) = \alpha$. Then $\underline{X} < \frac{1}{\alpha}\underline{X}$ and so $\gamma(\underline{X}) \leq \gamma(\frac{1}{\alpha}\underline{X})$. Since $\frac{1}{\alpha}\underline{X} \in \{0,1\}^n$, it follows from Proposition 2.3 that $\gamma(\frac{1}{\alpha}\underline{X}) = 1$. We claim that $\frac{1}{\alpha}\underline{X} \in P_1$.

Suppose, conversely, that $\frac{1}{\alpha}\underline{X} \notin P_1$. Since U_1 is closed, it follows from Proposition 2.1 that there exists a $\underline{W} < \frac{1}{\alpha}\underline{X}$ such that $\underline{W} \in P_1$. Consider the vector $\alpha\underline{W} \in \{0,\alpha\}^n$; it is easily seen that $\gamma(\alpha\underline{W}) = \alpha$ and thus there exists a vector $\alpha\underline{W} < \underline{X}$ such that $\alpha\underline{W} \in U_\alpha$. This contradicts the assumption that $\underline{X} \in P_\alpha$ and hence $\frac{1}{\alpha}\underline{X} \in P_1$ as claimed. This holds for all $\underline{X} \in P_\alpha$ and so $P_\alpha \subset \alpha P_1$.

Similarly, it can be shown that $\alpha P_1 \subset P_\alpha$. \square

3. THE CHARACTERIZATION THEOREMS

Theorem 3.1

A CSF γ is of the Barlow-Wu type if and only if it satisfies conditions C1, C2, C3 and C4.

Proof: It is easily verified that the Barlow-Wu CSF satisfies C1, C2, C3 and C4. To prove the converse, observe that

$$\begin{aligned} \gamma(\underline{X}) \geq \alpha &\iff \underline{X} \geq \underline{Y} \in P_\alpha \\ &\iff \min_{\{i \mid Y_i = \alpha\}} X_i \geq \alpha \text{ for some } \underline{Y} \in P_\alpha \\ &\iff \max_{\underline{Y} \in P_\alpha} \min_{\{i \mid Y_i = \alpha\}} X_i \geq \alpha \end{aligned}$$

$$\Leftrightarrow \max_{\tilde{Y} \in \alpha P_1} \min_{\{i | Y_i = \alpha\}} X_i \geq \alpha \text{ by Proposition 2.4}$$

$$\Leftrightarrow \max_{\tilde{Z} \in P_1} \min_{\{i | Z_i = 1\}} X_i \geq \alpha \text{ where } \tilde{Z} = \frac{1}{\alpha} \tilde{Y}.$$

This holds for all $\tilde{X} \in \Delta$ and $\alpha \in (0,1]$ and so

$$\gamma(\tilde{X}) = \max_{\tilde{Z} \in P_1} \min_{\{i | Z_i = 1\}} X_i.$$

Write $P_1 = \{\tilde{X}^{(1)}, \dots, \tilde{X}^{(N)}\}$ and let $T_j = \{i \in C | X_i^{(j)} = 1\}$. By the definition of P_1 , it is clear that each T_j is nonempty and that $T_j \not\subset T_k$ for all $j, k=1,2,\dots,N$ with $j \neq k$. Thus

$$\gamma(\tilde{X}) = \max_{1 \leq j \leq N} \min_{i \in T_j} X_i$$

where each $T_j \subset C$. Condition C4 ensures that $\bigcup_{j=1}^N T_j = C$, completing the proof. \square

Theorem 3.2

A CSF γ is of the Natvig type if and only if it satisfies C2 and

C1' γ is right-continuous

C4' For each $i \in C$ and all $\alpha \in (0,1]$, there exists an $\tilde{X} \in \Delta$ such that $\gamma(\alpha_i, \tilde{X}) \geq \alpha$ whereas $\gamma(\beta_i, \tilde{X}) < \alpha$ for all $\beta < \alpha$.

Proof: Baxter [3] proves that Natvig CSFs are right-continuous, and it is readily seen that such functions satisfy C2 and C4'. Conversely, from the preceding proof,

$$\gamma(\tilde{X}) \geq \alpha \iff \max_{Y \in P_\alpha} \min_{\{i | Y_i = \alpha\}} Z_{\alpha i} = 1$$

where $Z_{\alpha i}$ is the indicator of $\{X_i \geq \alpha\}$ ($0 < \alpha \leq 1$, $\tilde{X} \in \Delta$). Write $P_\alpha =$

$\{\tilde{X}^{(\alpha,1)}, \dots, \tilde{X}^{(\alpha, N(\alpha))}\}$ and let $T_j^\alpha = \{i \in C | X_i^{(\alpha,j)} = \alpha\}$, $j=1,2,\dots,N(\alpha)$.

Then $\gamma(\tilde{X}) \geq \alpha$ if and only if $\phi_\alpha(Z_\alpha) = 1$ where

$$\phi_\alpha(Z_\alpha) = \max_{1 \leq j \leq N(\alpha)} \min_{i \in T_j^\alpha} Z_{\alpha i}.$$

We claim that the binary functions $\{\phi_\alpha, 0 < \alpha \leq 1\}$ satisfy the conditions of the definition of the Natvig CSF.

It is clear that ϕ_α is nondecreasing in each argument for all $\alpha \in (0,1]$ and that $\phi_\alpha(Z_\alpha)$ is nonincreasing in α for fixed \tilde{X} .

To verify left-continuity, it is sufficient to consider the point at which the function decreases. Thus, suppose that $\gamma(\tilde{X}) = \alpha$ ($0 < \alpha < 1$); then there exists an $\tilde{X}' \leq \tilde{X}$ such that $\tilde{X}' \in P_\alpha$. Clearly, $\gamma(\tilde{X}') = \alpha$ and hence $\phi_\alpha(Z'_\alpha) = 1$ whereas, if $\beta > \alpha$, $\gamma(\tilde{X}') < \beta$ and so $\phi_\beta(Z'_\beta) = 0$. Thus $\phi_\alpha(Z_\alpha)$ is left-continuous as claimed.

Lastly, observe that, by C4', for each $i \in C$ and all $\alpha \in (0,1]$, there exists an $\tilde{X} \in \Delta$ such that $\phi_\alpha(1_i, Z_\alpha) = 1$ whereas $\phi_\alpha(0_i, Z_\alpha) = 0$ and so each ϕ_α is coherent.

This completes the proof. \square

4. SOME REMARKS ON THE BORGES-RODRIGUES CHARACTERIZATION

Let $S = \{0, 1, \dots, M\}$, $M \geq 1$. A nondecreasing mapping $\phi: S^n \rightarrow S$ with $\phi(\underline{0}) = 0$ and $\phi(\underline{M}) = M$ is said to be a multistate structure function (MSF). It is weakly coherent if $\max_{\underline{x} \in S^n} [\phi(M_i, \underline{x}) - \phi(0_i, \underline{x})] \geq 1$ for each $i \in C$.

If

$$\phi(\underline{x}) = \max_{1 \leq j \leq r} \min_{i \in P_j} x_i \quad (\underline{x} \in S^n)$$

where P_1, \dots, P_r are the r minimal path sets of a binary coherent structure function, then ϕ is said to be a Barlow-Wu MSF [1]. If $\phi(\underline{x}) \geq j$ if and only if $\phi_j(\underline{y}_j) = 1$ ($\underline{x} \in S^n$, $j=1, 2, \dots, M$) where $\{\phi_1, \dots, \phi_M\}$ is a collection of binary coherent structure functions such that $\phi_j(\underline{y}_j)$ is nonincreasing in j for fixed \underline{x} , and where y_{ji} is the indicator of $\{x_i \geq j\}$, then ϕ is said to be a Natvig MSF [6].

Borges and Rodrigues [5] present axiomatic characterizations of the Barlow-Wu and Natvig MSFs in terms of the following conditions:

- B1 For every $\underline{x} \in S^n$ with $\phi(\underline{x}) \geq k \geq 1$, there exists a $\underline{y} \in \{0, k\}^n$ such that $\underline{y} \leq \underline{x}$ and $\phi(\underline{y}) \geq k$
- B2 $\phi(\{0, M\}^n) = \{0, M\}$
- B3 ϕ is weakly coherent.

Borges and Rodrigues [5] claim

- (1) ϕ is a Barlow-Wu MSF if and only if it satisfies B1, B2 and B3
- (2) ϕ is a Natvig MSF if and only if it satisfies B1 and B3.

Both claims are false as the following examples attest.

Example 4.1

Consider the MSF $\phi_1: \{0,1,2\}^2 \rightarrow \{0,1,2\}$ defined as follows:

$$\begin{array}{lll} \phi_1(0,0) = 0 & \phi_1(0,1) = 0 & \phi_1(0,2) = 2 \\ \phi_1(1,0) = 0 & \phi_1(1,1) = 1 & \phi_1(1,2) = 2 \\ \phi_1(2,0) = 2 & \phi_1(2,1) = 2 & \phi_1(2,2) = 2. \end{array}$$

This satisfies B1, B2 and B3 and yet is clearly not of the Barlow-Wu type since the only Barlow-Wu MSFs of size two are $X_1 \wedge X_2$ and $X_1 \vee X_2$. Notice in particular that ϕ_1 provides a counter-example to Lemma 4 of [5].

Example 4.2

Let $\phi_1(Y_{11}, Y_{12}) = Y_{11}$ and $\phi_2(Y_{21}, Y_{22}) = Y_{21} \wedge Y_{22}$ and define the MSF $\phi_2: \{0,1,2\}^2 \rightarrow \{0,1,2\}$ as the function which satisfies $\phi_2(X_1, X_2) \geq j$ if and only if $\phi_j(Y_{j1}, Y_{j2}) = 1$ where Y_{ji} is the indicator of $\{X_i \geq j\}$ ($i, j, = 1, 2$). This is clearly not a Natvig MSF since the binary function ϕ_1 is not coherent, but it is easily verified that ϕ_2 satisfies B1 and B3.

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